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A probabilistic model for the persistence of early planar fabrics in polydeformed pelitic schists

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Abstract—Although early planar fabrics are commonly preserved within microlithons in low-grade pelites, in higher-grade (amphibolite facies) pelitic schists fabric regeneration often appears complete. Evidence for early fabrics may be preserved within porphyroblasts but, within the matrix, later deformation often appears to totally obliterate or reorient earlier fabrics. However, examination of several hundred Dalradian pelites from Connemara, western Ireland, reveals that preservation of early fabrics is by no means uncommon; relict matrix domains, although volumetrically insignificant, are remarkably persistent even when inferred later strains are very large and fabric regeneration appears, at first sight, complete.

Deterministic plasticity theories are ill-suited to the analysis of such an inhomogeneous material response, and a probabilistic model is proposed instead. It assumes that ductile polycrystal deformation is controlled by elementary flow units which can be activated once their associated stress barrier is overcome. Bulk flow propensity is related to the proportion of simultaneous activations, and a measure of this is derived from the probabilistic interaction between a stress-barrier spectrum and an internal stress spectrum (the latter determined by the external loading and the details of internal stress transfer). The spectra are modelled as Gaussian distributions although the treatment is very general and could be adapted for other distributions. Using the time rate of change of activation probability it is predicted that, initially, fabric development will be rapid but will then slow down dramatically even though stress increases at a constant rate. This highly non-linear response suggests that early fabrics persist because they comprise unfavourable distributions of stress-barriers which remain unregenerated at the time bulk stress is stabilized by steady-state flow. Relict domains will, however, bear the highest stress and are potential upper-bound palaeostress estimators. Some factors relevant to the micromechanical explanation of relict matrix domains are discussed.

INTRODUCTION

THE USE of microstructural relationships to infer the relative timing of deformation and metamorphic events is a well established procedure supported by a number of detailed reviews and many applications (e.g. Zwart 1960a,b, 1962, Johnson 1962, 1963, Spry 1963, 1969, Chadwick 1968, Vernon 1978). Some of the basic criteria used in such chronological analyses are illustrated in Fig. 1, from which it is clear that relict fabrics, whether preserved in porphyroblasts or within microlithons, are of central importance. It is unfortunately true that many of the 'standard patterns' used as the foundation for this type of analysis (continuity or otherwise of fabric elements, truncation relationships, fabric deflection adjacent to porphyroblasts, etc.) are not underwritten by detailed understanding of the mechanisms which contribute to them. Examples in which 'common-sense' interpretations of microstructural pattern are conflicting or erroneous have been discussed by Ferguson & Harte (1975), Vernon (1978) and Olesen (1983), among others. To a large extent these problems reflect our rudimentary understanding of the mechanisms by which fabrics develop and are subsequently modified or regenerated during later deformation.

The last remark applies especially to complex polymineralic rocks such as amphibolite facies pelitic schists. In contrast, our understanding of deformation mechanisms and fabric development in monomineralic rocks such as quartzite and limestone has advanced very rapidly over the last decade. It is entirely understandable that the research effort on micromechanisms of rock deformation should be concentrated initially on mineralogically simple systems. But field application of these advances is hindered by the disparity in mechanistic background available for these rocks compared with that in the pelitic to impure psammitic rocks which are volumetrically so important in many orogenic belts.

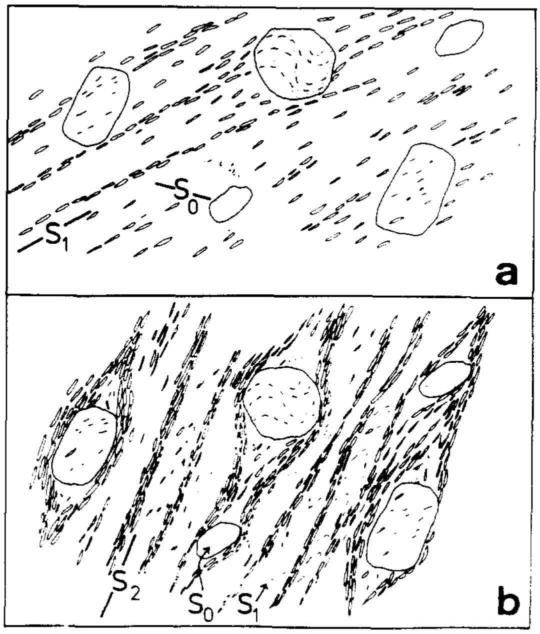

Fig. 1. Schematic diagram showing: (a) Preservation of bedding fabric, S_0, after development of first schistosity, S_1. (b) Relics of S_1 preserved within porphyroblasts and in microlithons after development of strong second schistosity, S_2; S_0 relics preserved only within some porphyroblasts. Adapted from Spry (1963, fig. 3).

Recent detailed work on slate microstructures (e.g. White & Knipe 1978, Knipe 1981) suggests that the final fabrics in, say, amphibolite facies pelites will reflect very complex distributions of stress, strain, deformation mechanisms, and metamorphic reactions together with the complexities introduced by high-temperature static recrystallization and grain growth. Furthermore, the experimental studies that have contributed so much to our understanding of quartzite and limestone deformation are hardly feasible for pelitic schists. It is therefore important that geologists working in metamorphic terrains are encouraged to recognize petrographic features which might have important micromechanical implications. For example, porphyroblasts may contain discordant inclusion fabrics (S_i being a relic of a fabric older than S_e) and yet may truncate S_e in the manner typical of post-S_e porphyroblasts (Ferguson & Harte 1975). Models of fabric regeneration that can account for such features are evidently needed!

This paper is concerned with another unexpected feature of fabrics in moderate- to high-grade rocks—the remarkable persistence of relict fabrics in the quartz-rich microlithons of pelitic schists. Thus, even when a planar fabric appears to be totally regenerated during a high-temperature, high-strain event, careful examination almost invariably reveals relics of earlier fabrics. The most satisfactory approach to this sort of problem would be to develop a deterministic micromechanical model for fabric regeneration in which the persistence of relict fabrics is a natural outcome. This seems a distant prospect. The treatment adopted in this paper, although motivated by micromechanical considerations, is instead fundamentally probabilistic. It is hoped that this approach, though necessarily very general in its formulation, will nevertheless help to focus attention on an aspect of microfabric development which might otherwise be overlooked. Before developing the model some examples of relict fabrics, and their geological setting, are outlined in the following section.

PLANAR FABRICS IN DALRADIAN PELITES, CONNEMARA

The orthotectonic Caledonides of Connemara (western Ireland) comprise late Precambrian to early Cambrian metasediments assigned to the Dalradian Supergroup (Harris & Pitcher 1975). They are characterized by intermediate-pressure amphibolite-facies metamorphism, and four major phases of ductile deformation. The geology is summarized by Tanner & Shackleton (1979) and Leake *et al.* (1981). The regional structure is dominated by major F_3 folds (some twenty of which have been mapped) which are arched over a major F_4 antiform (the Connemara Antiform) such that they are downward facing on the north limb of the F_4 structure and either upward or downward facing on the south limb depending on their relationship to major F_2 folds. The main penetrative fabric was formed during F_2. Although little quantitative strain estimation has been attempted in Connemara, extreme attenuation of the stratigraphic succession attests to very large strains, mostly accomplished during F_2. For example, the Bennabeola Quartzite Formation (the most prominent formation in Connemara both in terms of outcrop area and physiographic expression) is thinned to <1 m in several places and yet retains its stratigraphic integrity (Tanner & Shackleton 1979). Its original thickness is unknown; however, its stratigraphic equivalent in SW Scotland (Jura Quartzite) is about 5 km thick on Jura although this probably thins down to 0.5 km or so over a strike distance of less than 20 km to the northeast (Anderton 1976). In spite of the uncertainty introduced by such rapid changes in original thickness (which probably result from syn-depositional faulting), the combination in Connemara of intense fabric development and extreme stratigraphic attenuation leaves little doubt that the F_2 strains were very large, though difficult to quantify.

The syn-F_2 schistosity (S_2) in pelitic rocks is typically strongly differentiated into mica-rich laminae and quartz–plagioclase rich laminae. Pre-F_2 fabrics are sometimes preserved as inclusion trails within both garnet and plagioclase but relict S-surfaces preserved as discordant fabric elements within quartz-rich S_2 domains have not been widely reported from Connemara. However, it is the writer's experience that, although micas within quartz-rich S_2 domains are typically parallel to (and contribute towards) the S_2 schistosity, relics of an earlier fabric are by no means uncommon even when the F_2 strains are inferred to be very large.

In contrast to the S_2 schistosity, the appearance of fabrics developed in pelitic rocks during F_3 is very variable. In many areas it is a typical crenulation schistosity but in others a composite S_2/S_3 schistosity is found in which the syn-F_3 contribution appears to involve only rotation of S_2 (together with grain growth), but without recognisable micro-buckling of the S_2 fabric.

The preservation of relict fabrics is illustrated by examples from the Kylemore Formation (Fig. 2) which crops out along the northern margin of the Dalradian inlier in Connemara (see Leake *et al.* 1981, Cruse & Leake 1968). This is a particularly interesting area because here the S_3 fabric development is associated with the development of a major tectonic slide (the Renvyle–Bofin Slide, Fig. 2), probably of late-F_3 age. Kylemore Formation rocks on both sides of the slide exhibit greenschist facies assemblages characterized by a muscovite–chlorite schistosity and sparse porphyroblasts of chloritoid and garnet. But relics of staurolite and fibrolite attest to an earlier amphibolite-facies assemblage similar to that now found in the stratigraphically equivalent (?) Ballynakill Formation further south. The development of the Renvyle–Bofin Slide was probably associated with relatively rapid uplift producing F_3 structures and fabrics under greenschist-facies conditions while amphibolite-facies conditions persisted further south. The extent of fabric reconstruction is variable but locally on Renvyle, and throughout most of Inishbofin and Inishshark (Fig. 2), it is sufficiently

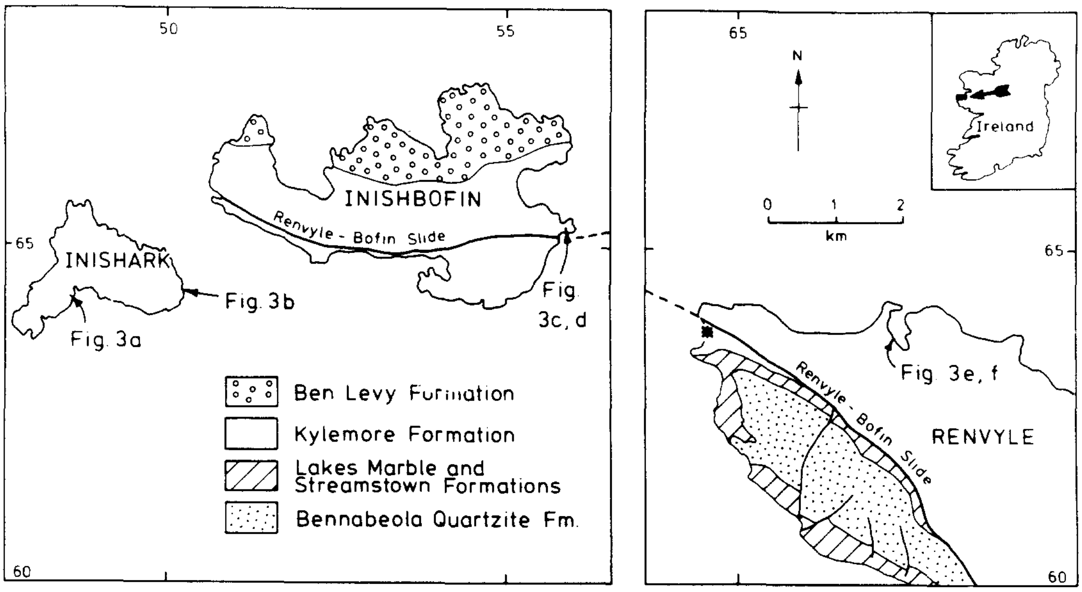

Fig. 2. Outline geological map of Inishark, Inishbofin and Renvyle peninsula, NW Connemara showing location of rocks illustrated in Fig. 3. Asterisk marks location of Kelly & Max (1979) finite strain estimate (see text). Map simplified from Leake *et al.* (1981).

intense for the pelitic rocks to take on a phyllonitic aspect.

The development of the S_3 schistosity in quartz–muscovite–chlorite schists is illustrated in Fig. 3. It initially develops as a crenulation cleavage with S_2 relics abundantly preserved in microlithons (Fig. 3a). Closer to the Renvyle–Bofin Slide, however, most phyllosilicates within quartz-rich domains are regenerated parallel to S_3; S_2 relict domains, although still clearly preserved (Fig. 3b) are uncommon. Very close to the slide the quartz-rich rocks are blastomylonites while the pelites are phyllonites showing extreme quartz-ribbon development in which quartz grains are either very elongate or are recrystallized to strings of quartz grains, mostly one ribbon width in size. Albites show optically continuous overgrowths on earlier cores, the overgrowths fingering along S_3 to produce extremely long grains. The net result of these processes is a very intense S_3 fabric. In thicker quartz-rich domains within the phyllonites, recrystallization has largely destroyed the strong quartz-shape fabric but micas show a very strong preferred orientation parallel to S_3 (Fig. 3c). But weak relics of the S_2 fabric still persist (Fig. 3d) although they are now very rare.

Although quartz–muscovite–chlorite phyllonites are locally well developed on Renvyle, over much of the peninsula the S_2 fabric still dominates although it is strongly crenulated by F_3 microfolds. The S_2 schistosity is typical of that throughout much of Connemara; it is strongly differentiated and phyllosilicates in the quartz-rich domains, although much smaller than those in mica domains, are parallel to, and contribute towards, the S_2 fabric (Fig. 3e). The strains involved in the D_2 event were probably of similar magnitude to those inferred elsewhere in Connemara. The only quantitative strain estimate from Renvyle was derived by Kelly & Max (1979) from pebble-rich schists on Renvyle Head (locality marked with asterisk in Fig. 2). The shape of the total strain ellipsoid is strongly oblate ($k = 0.06$), the ratios of the principal axes $X:Y:Z$ being estimated as 23:13:1. In spite of its proximity to the Renvyle–Bofin Slide this strain estimate mostly reflects the F_2 deformation, the F_3 strain resulting only in folding of pebbles already greatly flattened during F_2. Nevertheless, although inferred D_2 strains are large and the S_2 fabric always appears thoroughly penetrative, careful searching almost invariably reveals pre-F_2 relict fabrics (Fig. 3f).

The object of this paper is to help explain the remarkable persistence of relict matrix domains. In the next section some possible theoretical approaches are briefly reviewed in order to provide a comparative framework for the probabilistic approach finally adopted.

MECHANICS OF INHOMOGENEOUS DEFORMATION

The preservation of relict fabrics within porphyroblasts is commonplace because the porphyroblasts resist later yielding while the surrounding matrix readily deforms. Relict domains within the matrix persist because they too resist yielding relative to the surrounding matrix. However, many relict domains are similar in mineralogy and mineral proportions to nearby matrix domains in which the schistosity has been thoroughly regenerated to form a new schistosity. The basic theoretical problem posed by relict matrix domains is to account for this highly inhomogeneous response.

Deterministic approaches

Deterministic theories are not well-suited to this type of problem. Continuum theories of anisotropic plasticity hold out some promise because their relative simplicity allows them to be easily incorporated into finite-element programs. The price, of course, is loss of generality. A more important problem, however, is that the yield criteria and associated flow rules are postulated rather than derived. As yet there is little known about their applicability to foliated rocks (for example, are yield loci elliptical in plane stress as postulated?).

Theories of crystal plasticity such as the Taylor–Bishop–Hill model (Taylor 1938, Bishop & Hill 1951, see Lister *et al.* 1978 for application to quartzite) are more promising because the glide behaviour of individual grains is related to polycrystalline behaviour. At first sight the Taylor–Bishop–Hill analysis seems grossly ill-suited to the problem of relict fabric domains because it assumes that every grain undergoes the same shape change as the whole polycrystal. However, this in turn implies that the stress state will differ from grain to grain so that the approach could be adapted to the problem of modelling the relative resistance of different domains. But it remains uncertain whether the analysis (which assumes grain deformation solely by conservation motion of dislocations) could be developed to encompass more complex deformation mechanisms involving more than one mineral phase.

Perhaps the most promising deterministic approach will be found in the recent theories of plasticity which consider localized deformation as an instability in the macroscopic constitutive description of the material. This development was initiated by Hill (1958) and has been developed, and applied to geological problems, by Rice and his coworkers (Rice 1975, 1976, Stören & Rice 1975, Rudnicki & Rice 1976). In essence, the assumptions embodied in the classical flow-theory of plasticity (smooth yield surface and normality of plastic strain increments) are replaced by other assumptions (e.g. the development of a vertex-like structure on the yield locus) which lead to a bifurcation point in the deformation. The deformation is unstable in the sense that the constitutive relations allow homogeneous deformation of an initially uniform material up to a certain stress level beyond which localized deformation may occur; material outside the zone of localization continuing to deform homogeneously as before. This type of approach has obvious attractions especially as vertex formation on the yield surface of polycrystalline materials has been predicted theoretically (Hill 1967, Hutchinson 1970). However bifurcation models are much more complex than classical plasticity theories, and also require that values be specified for more parameters for which there is little or no relevant data for rocks.

In short, deterministic modelling of relict fabric preservation seems beyond current capability. It is therefore natural to explore the possibility of using a probabilistic approach.

Probabilistic approaches

This type of approach to the micromechanical modelling of deformation is exemplified by Feltham's (1973) stochastic model of dislocation kinetics in metals. The model assumes that creep rate is determined by a spectrum of energy barriers; an activated jump of a slip unit

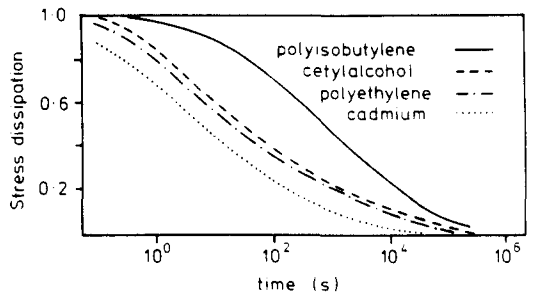

Fig. 4. Similar shape of stress–relaxation curves for solids of widely differing structure and composition. The organic solids are crystalline (cetylalcohol), semicrystalline (polyethylene) and amorphous (polyisobutylene); the cadmium is crystalline. After Kubát *et al.* (1979).

(e.g. a dislocation segment pinned at an obstacle) is assumed to take it from an energy barrier of height u_1 to a new one of height $u_2 = u_1 \pm \delta u$ where δu is a small fixed amount. The number of slip units with activation energy u is $n(u, t).\delta u$ per unit volume of crystal at time t. Feltham shows that, if each activated jump is assumed to make the same average contribution to the overall shear strain then the strain rate is proportional to the integral of the transition probabilities over the u-spectrum. This model provides a good representation of stress relaxation data and a rather less good representation of creep data (Bekirovic & Feltham 1974). Its relevance here is that it demonstrates the viability of replacing the detailed microphysics of dislocation motion with transition probabilities of an energy barrier spectrum.

The stochastic approach has been generalized by Kubát *et al.* (1979) who point out that creep and relaxation curves are very similar for widely different materials (Fig. 4). This suggests that macroscopic behaviour may be determined more by general interaction effects between elementary flow units than by their detailed microphysics. Whether the elementary flow units are dislocation segments in a mineral or macromolecular units in an organic polymer may not be of central importance in predicting bulk behaviour so long as their stochastic interactions can be modelled successfully.

Stochastic models of the type outlined above have not yet been developed for complex polymineralic materials. Nor do they explicitly deal with the inhomogeneous deformation that is central to the problem of relict fabric preservation. The treatment developed in the next section is therefore very different from the above models although substantially motivated by them.

A PROBABILISTIC MODEL FOR RELICT MATRIX DOMAINS

Micromechanical considerations

Although rock deformation is ultimately controlled by the velocity gradients associated with interacting lithospheric plates, the micromechanics of fabric development can be thought of as primarily stress driven. Local variations in stress are largely determined by the elastic properties of the various minerals and the

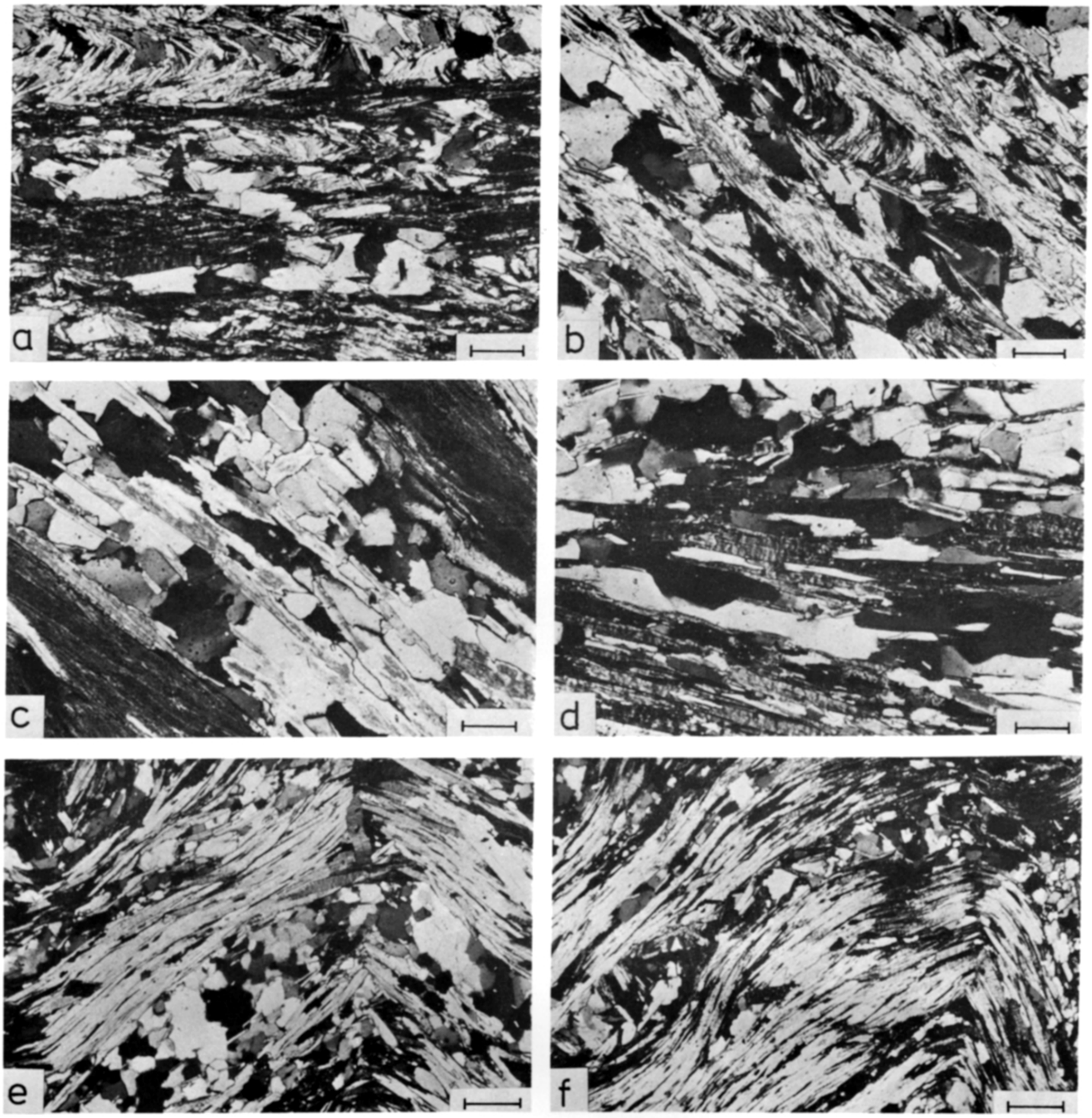

Fig. 3(a). Quartz–muscovite–chlorite schist, Inishshark, showing abundant S_2 relics within S_3 microlithons. (b) Quartz–muscovite–chlorite schist, Inishshark, in which most phyllosilicates have been regenerated parallel to S_3 with only occasional S_2 relics preserved. (c) and (d) Quartz-rich domains within muscovite–quarz–albite phyllonite, Inishbofin, showing typical preferred orientation of muscovite parallel to S_3 (c) and the rare preservation (d) of S_2 relics. (e) Strongly differentiated S_2 schistosity, with F_3 crenulations, in quartz–muscovite schist, Renvyle. Small muscovites within recrystallized quartz-rich domains are strongly parallel to S_2. (f) Pre-S_2 planar fabric preserved in very rare relict domain within S_2 schistosity at same locality as (c). Locations of specimens are shown on Fig. 2. Scale-bars are 100 μm (a)–(d), 300 μm (e) and 400 μm (f).

detailed geometry of grains, grain boundaries and intracrystalline defects. One can visualize a small volume of rock as comprising a large number of elementary flow units, any of which can be activated if its associated stress-dependent energy barrier is overcome. These stress barriers include: (1) the critical resolved shear stresses for the slip system of the various minerals; (2) the critical stress necessary to produce dislocation multiplication by, for example, Frank–Read sources and (3) stress thresholds associated with kinking and deformation twinning of grains. Kinking seems to be particularly important in controlling fabric reconstruction in pelitic tectonites (Etheridge & Hobbs 1974, Williams *et al.* 1977, White & Knipe 1978) and local resistance to kinking is probably a major factor in the preservation of relict matrix domains. In slates the lateral migration of new cleavage lamellae at the expense of relict lenticular domains seems to be the result of metamorphic reactions initiated by locally high stresses at domain margins (White & Knipe 1978); again a stress-dependent activation barrier is implied. Not all processes important in fabric development depend on local stresses reaching critical levels. Many diffusion processes (including dislocation climb) are stress driven but the stress acts in the same way as a concentration gradient in classical (Fick's law) diffusion; diffusion rate is proportional to stress but there is no threshold. But, except perhaps at very high homologous temperatures or very small grain sizes, the extent of fabric development will depend on the number of successful activations of the elementary flow units.

Let us now try to imagine a three-dimensional deterministic crystal plasticity model which could incorporate such micromechanical considerations. The easiest to visualize might be a finite-element type model in which the nodes represent elementary flow units (represented by their stress-barrier values) each of which is subjected to an internal applied stress determined by the external loading conditions and the mechanics of internal stress transfer. Considering all the nodes together, we have a distribution of stress-barrier magnitudes and a distribution of internal stress magnitudes. Of course, a finite-element type of approach is way beyond current capability: the nodal topology would need to be defined and the initial elements of the distributions would need to be in one-to-one correspondence. Nevertheless, progress is still possible if we are prepared to treat the distributions and their interaction in a probabilistic manner.

The model

In a small volume of rock comprising, say, a few hundred grains there will be a large number of stress barriers. It is therefore reasonable to treat the stress-barrier magnitudes as a continuous random variable: I will identify its probability density function (p.d.f.) by $R = g(r)$. Similarly, although stress is a second-order tensor varying from point to point throughout a volume of rock, I will consider a suitably resolved component of the internal stress to be a continuous random scalar variable with probability density function $T = h(\tau)$. The nature of these distributions in rocks is largely unknown. I will consider them to be normal (Gaussian) distributions only because, in the absence of relevant data, this seems intuitively reasonable. However, the treatment adopted here is very general and any pair of continuous density functions could be used. Nevertheless it needs to be stated that the equations in this and the next section have physical significance only to the extent that normal distributions (e.g. Fig. 5) are physically reasonable representations of the internal stress, and the resistance provided by stress barriers, in a deforming rock.

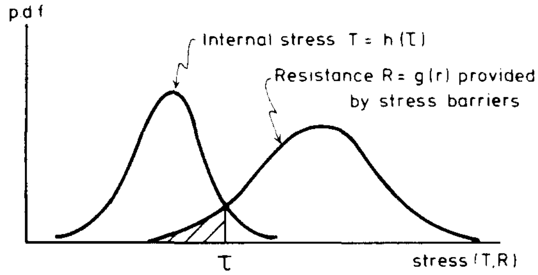

Fig. 5. Graphical illustration of conditional activation probability. Area of shaded region is given by equation (1) (see text).

I assume, then, that both R and T are normally distributed, with means M_R, M_T and standard deviations S_R, S_T respectively, and that $M_T < M_R$. Then, if τ is the internal stress applied at a stress barrier, the probability that τ is sufficiently large to activate the elementary flow unit is given by

$$\Pr(R \le \tau) = \frac{1}{S_R\sqrt{2\pi}} \int_{-\infty}^{\tau} \exp\left\{-\frac{1}{2}\left(\frac{r - M_R}{S_R}\right)^2\right\} dr \tag{1}$$

which is the shaded area on Fig. 5. The unconditional probability of activation, $P = \Pr(R \le T)$, is obtained by integrating eq. (1) over all possible realizations of T. Hence

$$P = \int \Pr(R \le \tau)h(\tau) \tag{2}$$

in which

$$h(\tau) = \frac{1}{S_T\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\tau - M_T}{S_T}\right)\right\}. \tag{3}$$

Equations (1) and (3) can be reduced to standard normal form using the substitutions

$$x = (\tau - M_R)/S_R,$$
$$y = (\tau - M_T)/S_T,$$

so that eq. (2) can be written

$$P = \int_{-\infty}^{\infty} F(x)f(y)\, d\tau/S_T \tag{4}$$

in which $F(\ )$ and $f(\ )$ are used to denote the standard normal cumulative density function (c.d.f.) and p.d.f., respectively. Then, if $a = S_T/S_R$ and $b = (M_T - M_R)/S_R$ we can write

$$x = ay + b$$

and eq. (4) becomes

$$P = \int_{-\infty}^{\infty} F(ay + b)f(y)\, dy. \tag{5}$$

After some further manipulation and substitution this can be written

$$P = \frac{1}{(a^2 + 1)^{1/2}\sqrt{2\pi}} \int_{-\infty}^{b} \exp\left\{-\frac{1}{2}\left(\frac{u^2}{a^2+1}\right)\right\} du \tag{6}$$

which is the cumulative distribution function of a normally distributed variable with mean equal to zero and variance equal to $a^2 + 1$. Thus in standard normal form we can write

$$P = F\left(\frac{b}{(a^2 + 1)^{1/2}}\right). \tag{7}$$

The probability of activation is therefore determined by the value of the ratio

$$\frac{M_T - M_R}{(S_T^2 + S_R^2)^{1/2}} = z. \tag{8}$$

For example, if the means and variances of R and T yield $z = -3.09$ then, from tables of the standard normal c.d.f., the probability of a successful activation is 0.001. Furthermore, at any given time the same analysis applies to each and every barrier so that $z = -3.09$ implies simultaneous activation of 0.1% of the elementary flow units. However, plastic deformation requires the co-operative activation of elementary flow units; successful activation of a few units results in local stress relaxation but the local displacements cause higher stresses to be transferred to neighbouring units thus inducing further activations, and so on. A very low z-value implies a low probability of co-operative activation. A moderately low value implies co-operative activation in domains where the lower-energy end of the stress-barrier spectrum is well represented, thus leading to incipient fabric development. As z increases so does the number of domains in which co-operative microflow occurs. As $z \to 0$ the number of simultaneous activations approaches 50% implying that only those domains with the most unfavourable disposition of stress-barrier will resist co-operative microflow (but see later). The z-value therefore provides a measure of bulk flow probability and also indicates the propensity for survival of relict matrix domains.

This probabilistic treatment is consistent with the spectacular inhomogeneity of microflow so clearly shown in deformation experiments using crystalline analogue materials (Means 1981, Tungatt & Humphreys 1981, Urai & Humphreys 1981, unpublished experiments of R. J. Knipe). This work demonstrates that microflow is initially very localized; but as stress increases (assuming work hardening) the volume fraction of material deforming at any one instant increases, although the domains involved continually switch as some 'lock-up' while others are newly activated or reactivated. It is not clear from these experiments whether or not the instantaneously deforming volume fraction reaches a steady state but it is reasonable to suppose that it will more or less stabilize as the steady-state flow stress is reached, even though the domains which contribute to that volume fraction will be different at different times. The implication for z-value behaviour is that z will increase with time as the stress increases but an upper limit will be reached governed by the steady-state flow stress.

Now it is clear that, without knowledge of the internal stress and stress-barrier distributions, a z-value cannot be calculated at any time. Nevertheless, the way in which z changes with time (in response, say, to increasing tectonic load) can be examined. Similarly if the variances of the posited density functions change with time, the effect on the behaviour of z can be studied.

TEMPORAL CHANGES IN ACTIVATION PROPENSITY

Equation (8) shows that activation probability is directly proportional to the difference between the mean internal stress and the mean stress-barrier magnitude. It will simplify what follows if this difference ($M_T - M_R$) is denoted by the symbol Δ. We now investigate how z will change with time as a function of temporal changes in Δ and/or the standard deviations of R and T. Writing eq. (8) as

$$z = \Delta(S_T^2 + S_R^2)^{-1/2}$$

and using the total differential

$$dz = \frac{\partial z}{\partial \Delta} d\Delta + \frac{\partial z}{\partial S_T} dS_T + \frac{\partial z}{\partial S_R} dS_R,$$

the time derivative of z becomes

$$\frac{dz}{dt} = \frac{d\Delta/dt}{(S_T^2 + S_R^2)^{1/2}} - \frac{\Delta}{(S_T^2 + S_R^2)^{3/2}}\left(S_T \frac{dS_T}{dt} + S_R \frac{dS_R}{dt}\right). \tag{9}$$

Some of the implications of this equation can be examined by using a few numerical examples to illustrate temporal changes in activation propensity. All the examples involve an increase in Δ with time and can therefore be interpreted in terms of increasing M_T (in response, say, to a progressive increase in tectonic loading), or decreasing M_R (in response, say, to increasing temperature), or some combination of these. In all the examples the initial conditions (at time $t = 0$) are $\Delta = -10$ MPa, $S_R = 1$ MPa, $S_T = 2$ MPa.

Model 1

In this example the dispersions of the R and T distributions do not change with time, $dS_R/dt = dS_T/dt = 0$. Accordingly dz/dt will vary directly with $d\Delta/dt$ and, if the means of the internal stress and stress-barrier distributions approach each other at a constant rate ($d\Delta/dt$ = constant), the time rate of change of z will also be constant,

$$dz/dt = (d\Delta/dt)(S_T^2 + S_R^2)^{-1/2} = \text{constant}.$$

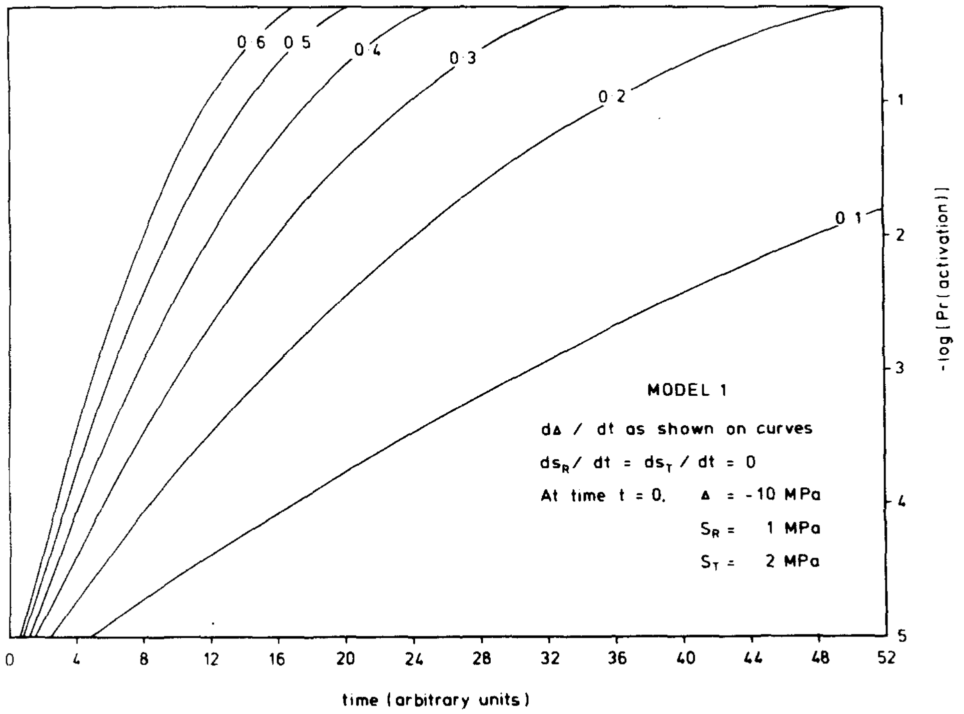

Fig. 6. Graph of activation probability (log scale) vs time for Model 1. Shapes of internal stress and stress-barrier distributions do not vary ($dS/dt = 0$) but distribution means approach each other at constant rate ($d\Delta/dt$ values shown on curves, see text). Initial conditions as shown in figure.

However the activation propensities, which are derived from successive values of z via tables of the standard normal c.d.f., do not increase at a constant rate (Fig. 6). The rate of increase depends, of course, on the value of $d\Delta/dt$ (as indicated on the curves, Fig. 6) but in all cases the rate of increase slackens off markedly with increasing time.

Model 2

The same range of $d\Delta/dt$ values used in Model 1 are again used here; but the dispersion of the R distribution now increases at a constant rate ($dS_R/dt = 0.2$) while the dispersion of the T distribution remains fixed as before. In this model dz/dt varies with time (see Fig. 7, curves

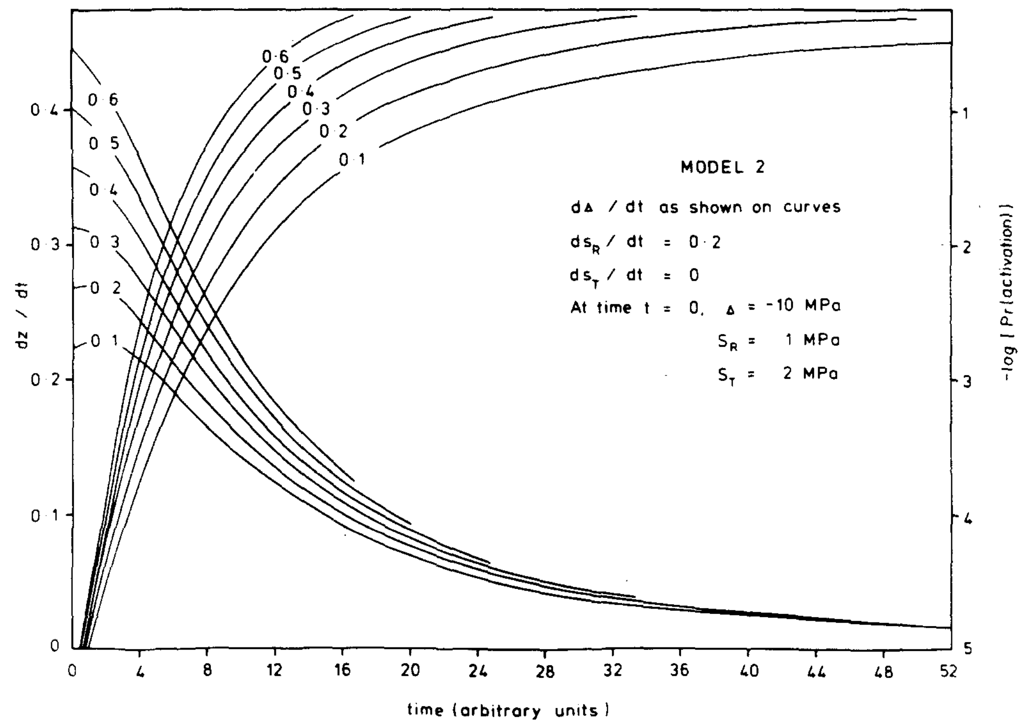

Fig. 7. Curves of dz/dt (labelled near left-hand ordinate), and activation probability (log scale) vs time for Model 2. Internal stress distribution is of constant shape; dispersion of stress-barrier distribution increases at constant rate ($dS_R/dt = 0.2$); curves are labelled with $d\Delta/dt$ values. Initial conditions as shown in figure.

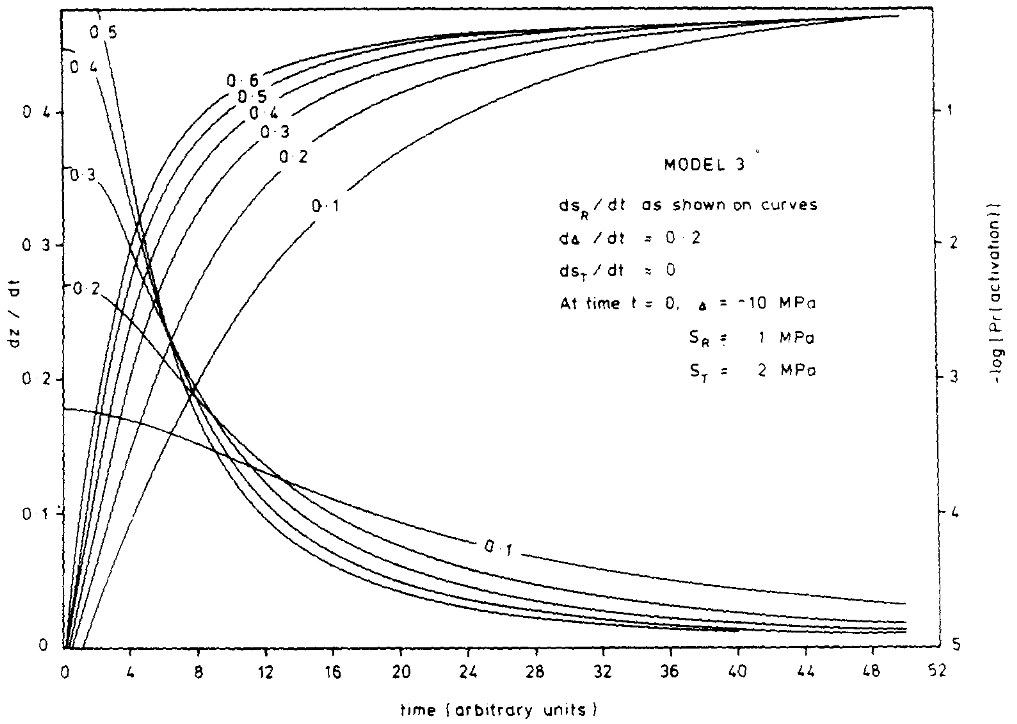

Fig. 8. Curves of dz/dt (labelled near left-hand ordinate), and activation probability (log scale) vs time for Model 3. Internal stress distribution is of constant shape; $d\Delta/dt$ is a constant (0.2 for all curves); curves are labelled with different dS_R/dt values. Initial conditions as shown in figure.

labelled near left-hand ordinate) and the resulting curves for activation probability show an initial steep increase with time, but start to flatten off rapidly once a probability of around 0.01 is reached. For example the $d\Delta/dt = 0.1$ curve (Fig. 7) exhibits a four orders of magnitude increase in activation probability in the first eleven time units but less than one order of magnitude increase in the next eleven units.

Model 3

This model illustrates the effect of one constant $d\Delta/dt$ in association with a variety of dS_R/dt values, the standard deviation of the internal stress distribution remaining fixed as before. The activation probability curves converge to $z = 0$ at a common time (Fig. 8) because the common $d\Delta/dt$ value implies that, irrespective of the rates at which S_R varies with time, the means of the R and T distributions will coincide at the same time. But the essential feature of Model 2, steep initial increase in activation probability followed by a rapid flattening off, is also clearly illustrated in Model 3.

DISCUSSION

Bulk deformation of a polycrystalline aggregate requires the co-operative activation of elementary flow units, both within and between grains. The activation probability derived in this paper is a measure of the proportion of elementary flow units activated at any one instant.

The models outlined in the previous section suggest that activation probability initially increases rather rapidly with increasing Δ (that is, with decreasing difference between the mean stress-barrier value and mean internal stress). The results of the three models are summarized in Fig. 9 in which the reciprocal of activation probability is plotted against time. The steep slopes of these curves suggest that, once microflow is initiated, fabric development will be relatively rapid with increasing Δ. However, all the curves show a dramatic flattening off once activation probability exceeds about 0.01, suggesting that domains with unfavourable distributions of energy barriers will be regenerated at a much slower rate even when Δ increases at a constant rate. Indeed, this flattening-off effect is so strong that it is likely to persist for internal stress and stress-barrier spectra that differ substantially from the Gaussian distributions assumed in this study.

Once co-operative microflow can take place in a sufficiently large volume fraction of rock such that a constant bulk strain rate is achieved, the internal stress distribution (and hence Δ) is likely to stabilize. Small volumes of rock with particularly unfavourable distributions of energy barriers will therefore persist as relict matrix domains. Some relict domains should therefore be expected, no matter how complete fabric regeneration may appear at first sight, and no matter how large the associated strain.

It is obvious that the most resistant (i.e. relict) matrix domains will bear the highest stress. However, an implication of the continuous distribution of stress-barrier magnitudes is that relict domains will represent combinations of stress barriers that are only slightly more unfavourable than those in other domains in which the schistosity has been regenerated. It follows that relict domains offer the potential for palaeostress estimation,

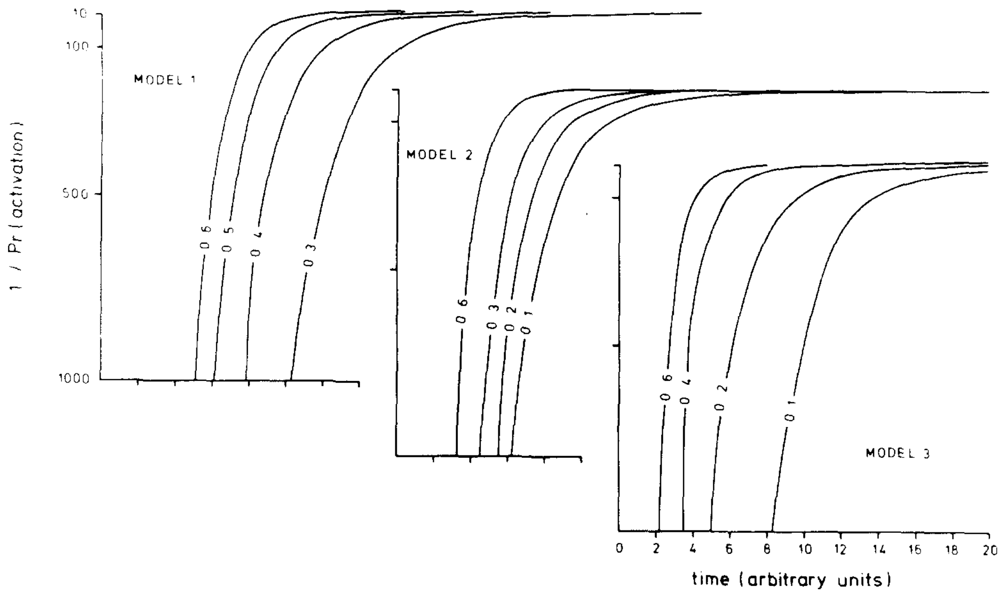

Fig. 9. Reciprocal of activation probability plotted against time for Models 1–3. The curves are labelled with $d\Delta/dt$ values (Models 1 and 2) or dS_R/dt values (Model 3).

for which they should form a close upper bound. Palaeostress analysis using such domains is likely to be difficult, however, because it entails calculating the external stress necessary to activate the relict domains.

This raises the problem of which factors, on the micromechanical level, are responsible for the preservation of particular relict domains. One obvious factor is the crystallographic orientation of the grains with respect to the principal stress axes. It is known, for example, that in some naturally deformed quartz-rich rocks the least deformed grains have *c*-axis orientations close to principal strain axes (see Fig. 10). Hence on the most active slip systems in quartz, (0001)⟨*a*⟩ and {10$\bar{1}$0}⟨*c*⟩, the resolved shear stress will be small. This crystallographic effect has also been reported in relatively undeformed 'augen' occurring in experimentally deformed quartzite (Tullis *et al.* 1973), although its interpretation in natural non-coaxial deformations (e.g. Bouchez 1977) is far from clear.

Another factor is that primary (Taylor–Bishop–Hill type) slip is expected to combine with secondary slip activated by misfit stresses where active primary slip planes end at grain boundaries. This idea is illustrated in Fig. 11 where it is envisaged that stress concentrations due to primary dislocation pile-ups at grain boundaries might lead to activation of slips systems (in the same or adjacent grains) that would otherwise be inactive. The suggestion, then, is that a Hall–Petch type grain-size effect might be combined with crystallographic orientation effects resulting in a small group of grains which largely resist deformation. Obviously when several minerals occur in a relict domain the micromechanical

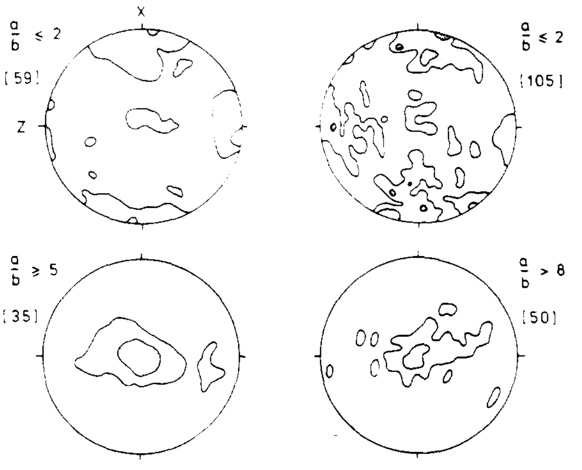

Fig. 10. Contoured lower hemisphere plots of *c*-axis orientations, referred to principal bulk strain axes $X > Y > Z$, for naturally deformed quartz grains. Plots for strongly deformed grains (large a/b values) show a maxiumum near Y. The least deformed grains (small a/b) show maxima around X, Y and Z with the dominant maximum around X. Simplified from Mancktelow (1981, fig. 3) and Bouchez (1977, fig. 13).

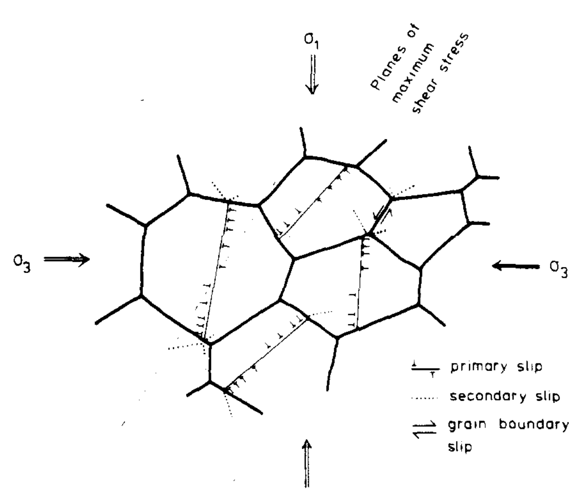

Fig. 11. Schematic diagram illustrating the possible influence of grain size on polycrystal deformation. In large grains which are favourably oriented for primary slip, large stress concentrations may develop where active slip planes end at grain boundaries, thus activating secondary slip on less favourably oriented slip systems in the same, or adjacent, grains.

complexity may be much greater; for example, the resistance to kinking of phyllosilicates will be combined in some way with the other factors outlined above.

It is hoped that this paper, by drawing attention to the micromechanical significance of relict domains, will stimulate the detailed analysis and modelling necessary to evaluate the palaeostress potential of relict fabrics.

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